

Contradiction, Consistency and the Paraconsistent
Perspective
in the Western Thought:
from Heraclitus of Ephesus to Newton da Costa

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paraconsistent logic

- the present state of **paraconsistent logic** attests to significant development, and its maturity permits a critical historical analysis of its development, having in view the appreciation of its **historical roots** and **stages of formation**.

paraconsistent logic

- our work attempts to discover how a truly **paraconsistent perspective** is constituted, as well as how logical principles, rules and systems have expressed the various **concepts of paraconsistency**.
- In this way, we may ask ourselves if logical principles and rules, according to which **not everything may be deduced from a contradiction, or something may be rejected**, were conceived and evoked within certain contexts and theoretical traditions.

paraconsistent logic

- the study of the logical meaning of **consistency** and **inconsistency** is found throughout the various periods of the history of philosophy, science and logic.
- diverse authors have investigated the **phenomena of contradiction**, seeking to identify, understand and neutralize its consequences for **rational knowledge**.

paraconsistent logic

analyzing the **historical precedents** of **paraconsistent logic** before the 20th century, we can identify some unanswered questions.

paraconsistent logic

- what ideas were proposed and debated with regard to **consistency** in that period of the history of formal logic?
- did such ideas **influence** later logical theories?
- was there knowledge of logical rules and principles which allowed, in some contexts, for inconsistency to be dealt without **trivialization**?
- if such principles were known, how were these proto-principles stated, and in what way can they be related to the **logical-paraconsistent results** and **rules** known **today**?

paraconsistent theories

- a logic is **paraconsistent** if it can be used as the underlying logic to inconsistent but non-trivial theories, which are called **paraconsistent theories**.

basic laws of the Aristotelian thought

- ▶ *Principle of (Non-) Contradiction*

$$\neg(A \wedge \neg A)$$

- ▶ *Principle of the Excluded Middle*

$$A \vee \neg A$$

- ▶ *Reflexivity of Identity*

$$\forall x(x = x)$$

Ex falso sequitur quodlibet

Ex impossibile sequitur quodlibet

(Principle of Explosion)

$$A \rightarrow (\neg A \rightarrow B)$$

$$(A \wedge \neg A) \rightarrow B$$

logical-paraconsistent elements in ancient authors



logical paraconsistent elements in ancient authors

contradiction and non-triviality from Heraclitus to Aristotle

- each in their own way, the contributions of **Heraclitus of Ephesus** and **Parmenides of Elea** are decisive for the later development of logic, particularly for having dealt with the question of **inconsistency** and **consistency**, and for having in a certain way placed the issue on the agenda of later philosophy, especially in the treatment of the theme by **Plato** and **Aristotle**.

logical paraconsistent elements in ancient authors

Heraclitus of Ephesus

(? 540- 470/80 BCE?)

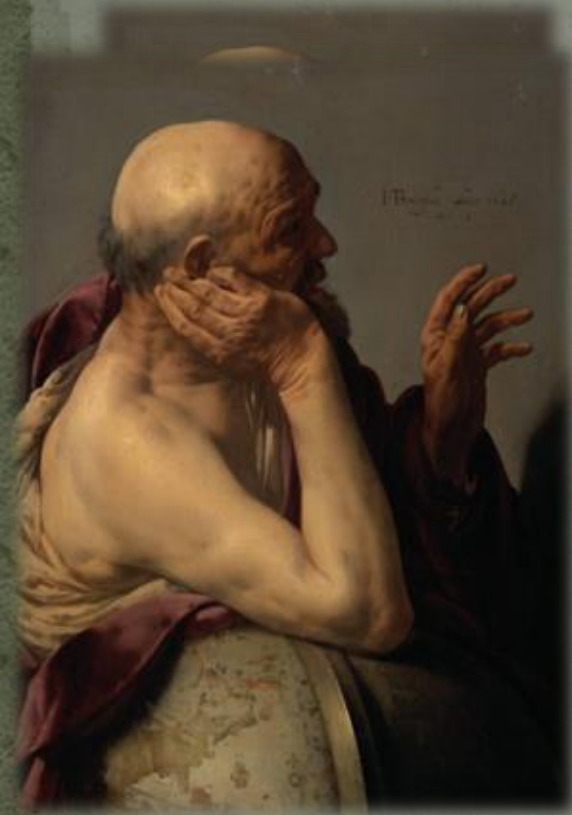
- **Heraclitus** writings are known to us only through fragments, which explain his **theory of the harmony of opposites** and are particularly important for the discussion of the annulment of the **principle of non-contradiction** (attributed by **Aristotle** to **Heraclitus**), and for their influence on later discussion in **ancient logic**.



Heraclitus
by Johannes Moreelse.

logical paraconsistent elements in ancient authors

Heraclitus of Ephesus (? 540- 470/80 BCE?)



- the **theory of the harmony of opposites** is in need of an interpretation which is more natural and in which the **identity of opposites** does not need to be denied.
- **Heraclitus'** ideas provoked a debate on the **study of contradiction** in rational knowledge and in logic.

Heraclitus
by H. der Bruggen

logical paraconsistent elements in ancient authors

Heraclitus of Ephesus (? 540- 470/80 BCE?)

- in relation to the **history of paraconsistent logic**, we may conclude that the ideas of **Heraclitus** can be seen as a rational system of explanation which, if formalized, can describe **contradictory states** without, at the same time, **trivializing** itself.

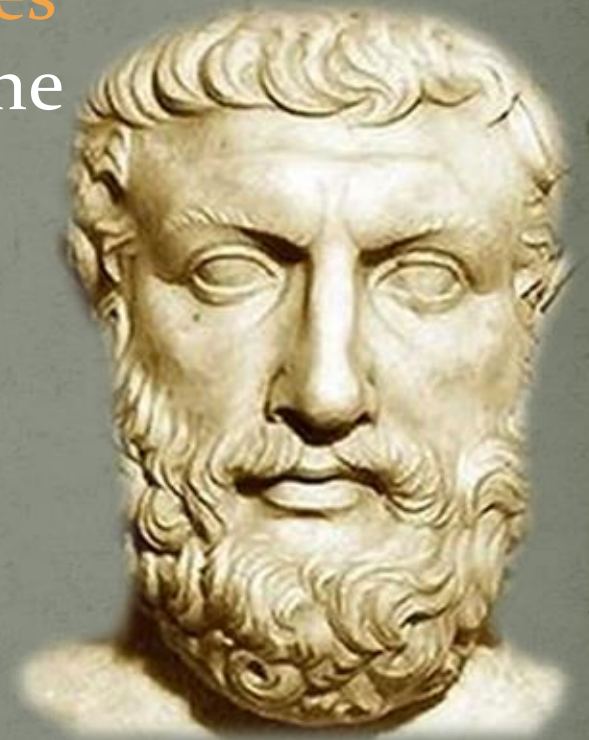


Democritus and Heraclitus
by Cornelis van Haarlem

logical paraconsistent elements in ancient authors

Parmenides of Elea (?515 – 449/40 BCE?)

- paradoxically, it is thanks **Parmenides'** contributions to **classical logic** that he is included in our work.
- his thought is known to us through fragments of the poem *On Nature*, where some may be interpreted as a **proto-enunciation** of the **three fundamental canons** of **classical deductive axiomatic thought**.

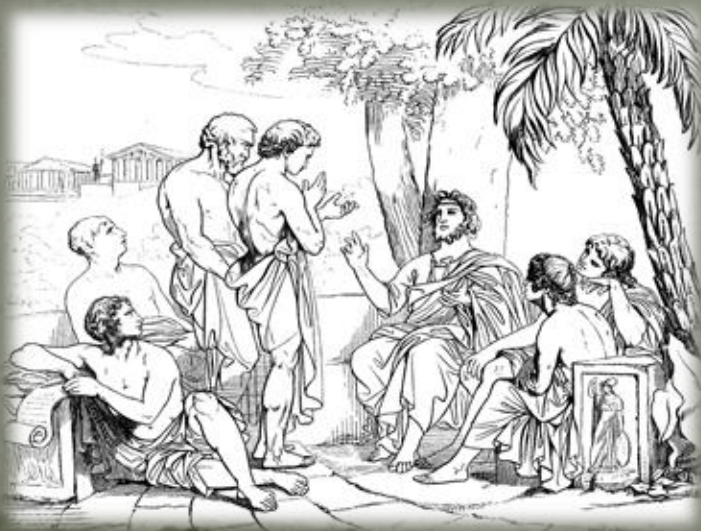


logical paraconsistent elements in ancient authors

Plato

(428-348/7 BCE)

- although the philosophical activity of **Plato** in one way or another took into account clearly **logical schemes of inference**, one cannot affirm that he studied logic for its own sake, as an autonomous form of knowledge.



Plato in his Academy
by Carl Johan Wahlbom

logical paraconsistent elements in ancient authors
contradiction and non-triviality in
Aristotle (384 – 322 BCE)

- the contributions of **Aristotle** to the **founding of logic** and of **scientific method** have been amply celebrated.
- we argue that in the core of his **theory of the syllogism**, he describes some deductive schemes in which the **presence of inconsistencies does not imply the trivialization** of the logical theory involved.



Aristotle, Opera Logica

logical paraconsistent elements in ancient authors
contradiction and non-triviality in
Aristotle (384 – 322 BCE)

- though not explicit, the notion that **Aristotle** proposed results of a **paraconsistent character** is corroborated by various theoretical situations he examined. In *Prior Analytics* (B15 63b 22–64b 27), he explains how valid syllogisms based on opposite (contrary or contradictory) premises can be obtained.



Marble bust of Aristotle.
Roman copy of a Greek
bronze, 330 BC

logical paraconsistent elements in ancient authors
contradiction and non-triviality in
Aristotle (384 – 322 BCE)

Aristotle further deepens his analysis of the **sylogistic consequence** by stating, in the *Prior Analytics*:

- *A true conclusion may be derived from false premises (B2)*
- *From opposite premises (contrary or contradictory) a valid (negative) conclusion may be derived in specific moods of the second and third figures (B15).*

logical paraconsistent elements in ancient authors
contradiction and non-triviality in
Aristotle (384 – 322 BCE)

these results are the basis for some of the rules for the **evaluation of valid syllogisms**, justify some *consequentiae* in medieval logic, and are at the center of the debate on the *ex falso sequitur quodlibet*.

logical paraconsistent elements in ancient authors

valid syllogisms based on
opposite premises

- recently, based on the same excerpt, Priest (2005) affirms that the syllogistic is paraconsistent.
- the earliest suggestion of this is that of da Costa and Bueno (1998).

logical paraconsistent elements in ancient authors

the scientific syllogism with inconsistent terms

- in Chapter 11 of Book A of *Posterior Analytics*, Aristotle shows that the principle of non-contradiction is not a general presupposition for any demonstration whatever, but only for those in which the conclusion must be proved on its basis.

Book A of the *Posterior Analytics*



ΑΡΙΣΤΟΤΕΛΟΥΣ ΑΝΑΛΥΤΙΚΩΝ ΎΣΤΕ
ΡΩΝ ΗΤΟΙ ΤΗΣ ΑΓΡΟΔΕΙΚΤΙΚΗΣ
ΠΡΩΤΟΝ.



Α Σ Α Διδασκαλία καὶ πᾶσι μα-
θηταῖς διδρακτικῆ, ἐκ προῦπερ χεῦσε
τίμητ' γνώσεως. Φανερὸν δὲ τὸ ἴδιον
ροῦσιν ἐπὶ πασῶν. Αἰτιῶν μαθημα-
τικῶν ἢ ἐπισημῶν, διὰ τὸ ἴδιον τὸ
πᾶσι περὶ αὐτῶν καὶ ἢ αἰώνια καὶ
τελευτῶν ἢ ἐπισημῶν ἢ αἰώνια καὶ

logical paraconsistent elements in ancient authors

contradiction and non-triviality in Aristotle

- Gomes and D'Ottaviano (2010) showed that it is possible to interpret the **Aristotelian demonstration** in the *Posterior Analytics* (A11) in contemporary terms, formalizing it in da Costa's paraconsistent logic C_1^*

GOMES, E. L., D'OTTAVIANO, I. M. L. (2010). Aristotle's Theory of Deduction and Paraconsistency. *Principia: International Journal of Epistemology*, vol. 14 (1). p. 71-97.

logical paraconsistent elements in ancient authors

the scientific syllogism with
inconsistent terms

- it seems possible to interpret the syllogisms on the basis of opposite premises, as in a broad paraconsistent theory.

logical paraconsistent elements in ancient authors

the scientific syllogism with inconsistent terms

- we suggest that the **role** of **Aristotle** in the **pre-history of paraconsistent logic** seems to be much more important than is customarily admitted.



Aristotle contemplates
Homero's bust
by Rembrandt, 1653

logical-paraconsistent elements in medieval authors



logical-paraconsistent elements in medieval authors

we present, based on some **key authors** of the **medieval period**, considerations and results related to **contradiction** and the *ex falso sequitur quodlibet* that are pertinent to a **history of paraconsistent logic**.

logical-paraconsistent elements in medieval authors

Boethius and the *ex falso*

(?475-7 – 526?)



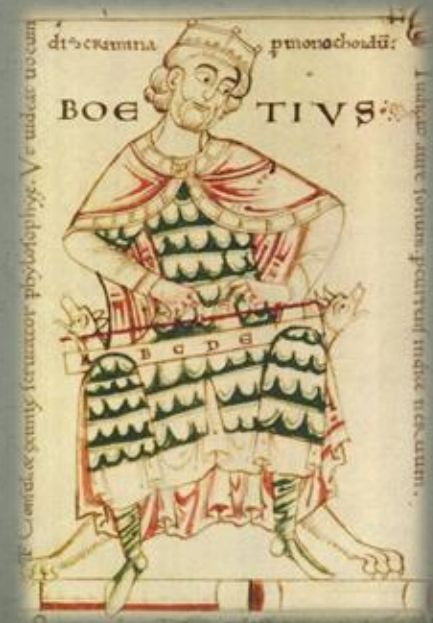
- **Boethius** left seminal elements for the various positions **for and against the *ex falso*** that are found in the scholastic phase of medieval logic.
- **Boethius** attracted the attention because of a passage that was very important in the conceptual construction of the ***ex falso***.

logical-paraconsistent elements in medieval authors

Boethius and the *ex falso*

(?475-7 – 526?)

- **Boethius** concisely describes **Artistotle**'s steps in the first chapters of **Book B of *Prior Analytics***, analyzing the case in which a syllogism can have two false premises and a true conclusion.



logical-paraconsistent elements in medieval authors
scholastic logic and thought

- one sees in the era of Scholastic logic an intense **debate** on the **validity of the *ex falso*** at the center of the **doctrine of topics, theories of implication, and duties disputes.**

logical-paraconsistent elements in medieval authors opposition to the *ex falso* in scholastic logic

- the first writers to mention the *ex falso sequitur quodlibet* or *ex impossibile sequitur quodlibet* were Garlando Compotista (11th century) and Peter Abelard.
- from the 13th century, there was intense debate concerning the **validity** of some *consequentiae* that were later accepted, and one finds the quarrel over the **legitimacy of the *ex falso***.

logical-paraconsistent elements in medieval authors
opposition to the *ex falso* in scholastic
logic: Peter Abelard

(1079 – 1142)

- it is in the context of **topical inferences** and **maximal propositions** that the rejection of the *ex falso* by **Abelard** is evident.

Statue of Abelard
at Louvre Palace in Paris
by Jules Cavelier



logical-paraconsistent elements in medieval authors
opposition to the *ex falso* in scholastic
logic: Peter Abelard

(1079 – 1142)

- his **topical investigation** leads **Abelard** to propose **new semantic criteria** for the notion of **necessary consequence** (*consecutio necessitas*), understood by some medieval authors in the manner of contemporary **material implication**.

logical-paraconsistent elements in medieval authors
**opposition to the *ex falso* in scholastic
logic: Peter Abelard**

(1079 – 1142)

- **Abelard** suggests that a **stricter notion of consecutio** is necessary:

The antecedent of a true declarative conditional sentence requires the consequent intrinsically.

logical-paraconsistent elements in medieval authors
opposition to the *ex falso* in scholastic
logic: Peter Abelard

(1079 – 1142)

- for **Abelard**, the *ex falso* is not valid, and from the false and from the impossible any consequent whatever does *not* truly follow. Relevant and paraconsistent elements seem to orient **Abelard**'s logical options.



Abelard and Héloïse
Manuscript *Roman de la Rose*
14th Century

logical-paraconsistent elements in medieval authors
more opponents of the *ex falso*

- some **other medieval authors** of the **13th century** objected to the *ex falso*.
- the veto on the *ex falso* can be placed within the panorama of a **metaphysical-logical-epistemological** approach of a **paraconsistent character**.

logical-paraconsistent elements in medieval authors
more opponents of the *ex falso*

Robert of Melun – 1100-1167



Petrus Hispanus
1266-1308



William of Ockham
1285-1347



Henry of Ghent
1217-1293

logical-paraconsistent elements in medieval authors
positions favorable to the *ex falso* in
scholastic logic: Adam of Balsham
(?–1159)

- John of Salisbury (1115-1120 – 1180) attributes to Adam of Balsham the thesis that “from a contradiction follows the same” (*idem esse ex contradictione*), a thesis which implies the *ex falso*, which generalizes it.



Adam of Balsham

positions favorable to the *ex falso* in scholastic logic

Duns Scotus and the logical-classical founding of the *ex-falso* (1266 – 1308)



- at the beginning of the 20th century, the *ex falso* was attributed to **John Duns Scotus**, because a clear and proficient statement of this logical law is found in two of his *commentaries*.

John Duns Scotus
15th century , by Justus van Gent

positions favorable to the ex falso in scholastic logic
Duns Scotus and the logical-classical
founding of the *ex-falso* (1266 – 1308)

- however, based on an examination of the material initiated around 1936 by Longpre, these works were attributed to another author, a 'Pseudo-Scotus'.
- this new viewpoint was sanctioned by the Vatican edition of Scotus' *Opera Omnia*, that began to be published in 1950.

positions favorable to the ex falso in scholastic logic

Duns Scotus and the logical-classical founding of the *ex falso* (1266 – 1308)

- this understanding of the issue has prevailed to the present day. Łukasiewicz (1951), not having taken into account these revisions in his celebrated study of the *Aristotelian syllogism*, promulgated the hypothesis that the *ex falso*, that he called *The Law of Duns Scotus*, should be attributed to the *Doctor Subtilis*.
- due to the prestige of the great Polish logician and historian, this eponymous solution was replicated by numerous other scholars.

positions favorable to the ex falso in scholastic logic

Pseudo-Scotus and the logical-classical founding of the *ex-falso*

- **Pseudo-Scotus** is one of the most intriguing personalities in the **history of logic**. Wise and talented, he developed in an extremely elegant way various **logical theories** typical of his period

positions favorable to the ex falso in scholastic logic

Pseudo-Scotus and the logical-classical founding of the *ex-falso*

- although **Pseudo-Scotus** was not the first to enunciate the *ex falso*, was for some time during our era admitted as such.
- the **argumentation of the author** explains in part the merit achieved by this exposition. Aside from the *ex falso*, some of its **corollaries** are stated in his conclusions.

positions favorable to the ex falso in scholastic logic
the logical-classical founding of the *ex-falso*

- as it may be seen, at this point in the development of medieval logic the *ex falso* had been completely integrated into the well-developed theory of the *consequentiae* as we find it in Pseudo-Scotus.
- the elegance of his treatment attests to the very mature development of the logical-classical paradigm, in this case in detriment to other perspectives such as those of relevance and paraconsistency.

logical-paraconsistent elements in medieval authors **considerations**

- to the extent that **logical-classical citizenship** may be conceded to the **rule**, there is a logical-classical solution to the conflict.
- on the other hand, in **rejecting** the *ex falso* and the logical-classical solution to the phenomenon of contradiction, various authors have **delineated an alternative approach** which encompasses elements pertinent to the **paradigm** now known as **paraconsistent**.

logical-paraconsistent elements in medieval authors **considerations**

- **modern authors** are more easily brought to **accept** as decisive the **methodological criterion of logical-classical coinage**, according to which any inconsistencies or contradictions inevitably bring the rational theory in which they occur to falsity (triviality).
- this predominance in the modern period will only be definitively reconsidered with the advent of **contemporary paraconsistency**.

contemporary paraconsistency



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contemporary paraconsistency

- the context of logic in the **modern era** is, in fact, philosophically complex and formally poor.
- the sparse logical-formal elements cultivated in **16th and 17th Europe** reflect the distinct **Medieval** logical traditions.

the analysis of inconsistency by some modern authors

- modern scholars famous for their contributions to philosophy, incorporate very little formal logic in their theoretical elaborations.

the analysis of inconsistency by some modern authors
the role of inconsistency in some of Leibniz'
logical theories
the rationalist system of Christian Wolff



Leibniz
(1646 – 1716)



Christian Wolff
(1679 – 1754)

the analysis of inconsistency by some modern authors

the apagogic strategy in Hume

Kant apagogic methods, and non-contradiction



Immanuel Kant
(1724 – 1804)



David Hume
(1711 – 1776)

the analysis of inconsistency by some modern authors
**the role of contradiction in the
speculative philosophy of Hegel**
(1770 – 1831)

- we call attention to **Hegel**'s position and contribution to rehabilitating the role of **contradiction** in **knowledge**, reopening the philosophic trail so that other theoreticians could seriously consider the role of contradiction in **broad rational contexts**.



Georg Hegel

on the trail of contemporary logic

- at the beginning of the 20th century, particular importance must be attributed to the rebirth of the study of logic, metaphysics and ontology, the foundations of mathematics and science specially in Central Europe.

on the trail of contemporary logic

- the mathematical environment of the 19th century is notable, above all, for the advent of non-Euclidean geometries, motivating a similar attitude in logic and facilitating the creative freedom that is so characteristic of logical contributions of the 20th century, which have shown themselves to be extremely fruitful, especially in relation to non-classical logic.

on the trail of contemporary logic

Jan Łukasiewicz and his project of a non-Aristotelian logic

(1878 – 1956)

- **Jan Łukasiewicz** is one of the great names of contemporary logic and the importance of his contributions is recognized by various scholars of the present day, in **logic** as well as in **philosophy**.



Jan Łukasiewicz

on the trail of contemporary logic

Jan Łukasiewicz and his project of a non-Aristotelian logic (1878 – 1956)



Jan Łukasiewicz, 1935

- Łukasiewicz employs his analysis of the principle of non-contradiction and ends by concluding that it is logically dispensable.
- in this sense, the Polish scholar clearly sees the path to a project for non-Aristotelian logics in which the latter principle no longer holds.

on the trail of contemporary logic

Jan Łukasiewicz and his project of a non-Aristotelian logic

(1878 – 1956)

- his conclusions directly influenced the appearance of the first **paraconsistent logical systems**, especially within the **Polish school of logic**.

on the trail of contemporary logic

Vasiliev and the plan for non-Aristotelian logics (1880-1940)

- **Nicolai A. Vasiliev** defended a **bold non-classical logical-theoretical project** with original ideas and suggestions. In 1912, he outlines an explicit project of **alternative heterodox non-classical logics**. His ideas united the **paraconsistent, many-valued, and intensional** approaches.



Nicolai Vasiliev

on the trail of contemporary logic
final considerations

- other scholars will realize the plans of Łukasiewicz and Vasiliev.
- the mathematical milieu of the 19th century and the advent of mathematical logic at the beginning of the 20th century, with its appropriate tools, made these steps possible, firm, and successful.

paraconsistent logic

- as a consequence of our historiographic premises, we consider **Stanisław Jaśkowski** (1906-1965) and **Newton da Costa** (1929-) the creators of paraconsistent logic. Motivated by problems arising from the presence of **contradictions** in specific rational contexts, they proposed and developed **axiomatic logical systems** capable of dealing with **contradictions** and **inconsistencies**, without a **trivialization** of the implied theories.

paraconsistent logic

Stanisław Jaśkowski

- **Stanisław Jaśkowski** (1906 – 1965), motivated by **Hegelian** and **Marxist** interpretations of contradiction, introduced (1948, 1949) the logic **D₂**, which **tolerates contradictions**. The motivation for this logic derives from the fact that the presence of contradictory statements in ordinary language is common, and the use of contradictory hypotheses is often necessary for the explanation of phenomena in scientific theories.



Stanisław Jaśkowski

paraconsistent logic

Newton da Costa

- **Newton da Costa** (1963) is very clear in proposing his hierarquies of paraconsistent logics, as alternative axiomatic logical systems, developing as well logics of a higher order that are able to overcome the limitations that contradictions impose on rational theories in the logical-classical paradigm.

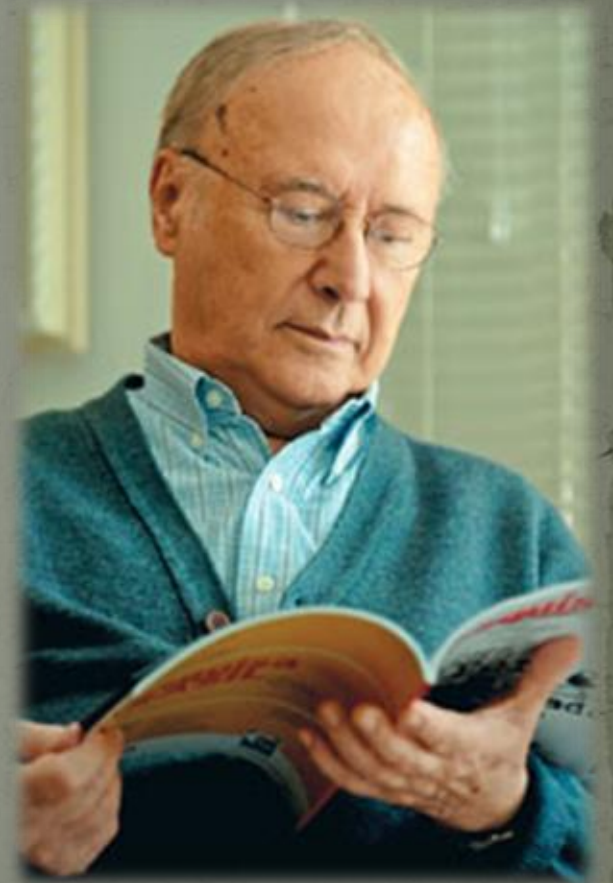
Newton da Costa



paraconsistent logic

Newton da Costa

- In this context, aside from the fact that **da Costa** and **Jaśkowski** fit the **criteria of intentional and semantic paraconsistency**, the contributions
- of these two authors called the attention of the community of logicians to a **new investigative program** which delineated, little by little, the shape of the **present-day field of paraconsistent logic**.



Newton da Costa

paraconsistent logic

final considerations

- The initial proposals for **paraconsistent systems** encouraged many **scholars** to study **paraconsistency** in its variety of forms, including those arising from **relevance logics**, **modal logics**, **fuzzy logics** and others, and these have been pursued by researchers of various nationalities and continents, especially in **Belgium**, **Australia**, **Italy**, **Russia**, **Israel** and the **United States**.

paraconsistent logic

final considerations

da Costa, his disciples and collaborators from several countries, have introduced many paraconsistent systems and obtained relevant results concerning algebraic structures associated to such systems, paraconsistent set theories, model theory, logics of higher order, paraconsistent differential calculus, and some applications to theories based on semantically closed languages, ethics, other non-classical logics, theory of probability, foundations of the infinitesimal calculus and of quantum mechanics, cognitive sciences, computer science, translations and combinations of logics.

Newton da Costa and the Brazilian School of Logic

paraconsistency has become a field of knowledge, inter-relating distinct schools of paraconsistent logic, and there are applications of paraconsistent logic not only to the foundations of science and its philosophical analysis, but even to informatics and technology.

the **Brazilian School of Logic**

Arquivos Históricos - CLE



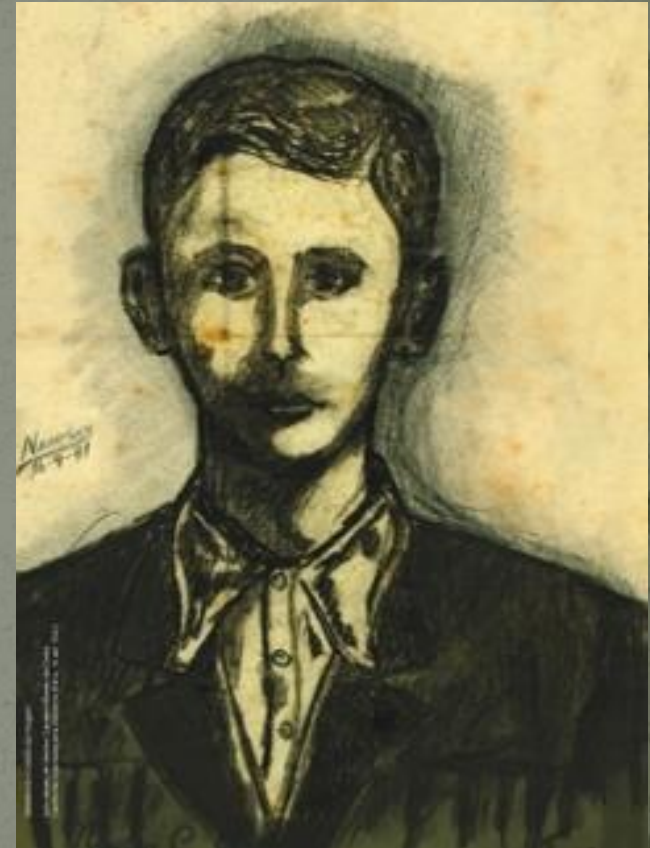
Ayda Ighes Arruda



paraconsistent logic

final considerations

- it is precisely from this **perspective** that we have studied the initial and general development of **paraconsistent logic** with an **emphasis** on the **history of Newton da Costa's paraconsistent systems**, as well as his contributions to the inauguration of this **field of logic** in the **twentieth century**.



Newton da Costa
Self-Portrait - 1944

our recent book

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- publisher:

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Coleção CLE, vol. 80

- 712 pages



the title

- *“Para além das Colunas de Hércules, uma história da paraconsistência: de Heráclito a Newton da Costa”*
“Beyond the Columns of Hercules, a History of Paraconsistency: from Heraclitus to Newton da Costa”
- the title is an allusion to the epic metaphor of surpassing limits, such as the founders of paraconsistency have made transcending classical limits of logicity.

further work

- *Illuminating Contradiction:
A History of Paraconsistency
from Heraclitus of Ephesus
to Newton da Costa*



obrigada!

спасибо!

thank you!