INVARIANCE
AS A BASIS FOR
NECESSITY & LAWS

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INTRODUCTION

MANY PHILOSOPHERS FIND NECESSITY BAFFLING.

HUMEANS, IN PARTICULAR, ARE DEEPLY DISTURBED BY THE THOUGHT OF NECESSARY LAWS OF NATURE.

THEY VIEW THE IDEA OF SUCH LAWS AS AN ARCANE IDEA OF SECRET, INEXPLICABLE, MYSTERIOUS POWERS GOVERNING THE WORLD.

IN THIS TALK I OFFER A DOWN-TO-EARTH YET SYSTEMATIC EXPLANATION OF NECESSITY & LAWS IN TERMS OF INVARIANCE.

THE IDEA OF INVARIANCE

INVARIANCE IS AN IMPORTANT AND HIGHLY EXPLANATORY NOTION, widely used in all branches of KN, from Logic to Physics.

IN THIS TALK I REGARD INVARIANCE AS A RELATION:

X is INVARIANT under Y.

I FOCUS ON A TYPE OF INVARIANCE THAT IS APPLICABLE IN ALL FIELDS OF KNOWLEDGE.

ROUGHLY:

PROPERTY X IS INVARIANT UNDER REPLACEMENTS OF INDIVIDUALS, Y.

UNDERLYING IDEA

Properties in general are *selective*. They "pay attention" or "are attuned" to *some* features of objects but not to others.

Accordingly, they distinguish between some objects but not others.

For ex:

The property *has-a-mass* distinguishes between stones and numbers (it applies to the former but not to the latter), but it does not distinguish between stones and stars (it applies to both).

The same applies to the property *is-subject-to-gravity* or *partakes-in-gravity*.

Selectivity as INV

This can be expressed in terms of INV:

The property *partakes-in-gravity*
is INVARIANT UNDER all
1-1 (and Onto) REPLACEMENTS of
stones by stars.

But:
It is NOT INV under any
1-1 (and Onto) REPLACEMENT of
stones by numbers.

Cont.

Another ways to put this:

If you replace stones by stars,
you replace some objects that
partake in gravity
by other objects that
partake in gravity.

The property *partakes-in-gravity* is not affected.

It is INV under these replacements.

Cont.

But:

If you replace stones by numbers,
you replace objects that
partake in gravity
by objects that
don't partake in gravity.

The property *partakes-in-gravity*
is NOT INV under such replacements.

Cont.

The selectivity of properties introduces structure – INV Structure – into the world.

Some properties are more selective than others – have a HIGHER DEGREE of INV – and different properties are selective in different ways – are INV under different replacements of individuals.

Cont.

But to understand the selectivity of properties we have to take into account not just ACTUAL but also COUNTERFACTUAL individuals.

Exs.

1. **Co-extensional Properties: *has-a-heart* & *has-a-kidney.***

These properties are INV under the SAME r's of ACTUAL individuals. But NOT under the same r's of COUNTERFACTUAL individuals.

2. **Regular Properties: *partakes-in-gravity.***

We don't understand the pattern of selectivity (INV) of *partakes-in-gravity* if we don't realize that if Earth had another Moon, it would *partake in gravity* as much as its actual moon. *Partakes-in-gravity would* not distinguish bet_ them.

Cont.

This notion of Counterfactual Individual is NOT a Philosophical Term-of-Art. And it is not associated with any Specific Philosophical Theory of Counterfactuals.

It is the EVERYDAY, PRETHEORETIC notion that we use in mundane discourse as well as in all Sciences.

(Would Medicine M have been effective, had <this or that> been the case?)

It is a common notion, and there is nothing mysterious about it.

CLAIMS:

I. **EVERY PROPERTY IS** INVARIANT **UNDER SOME 1-1 AND ONTO REPLACEMENTS** r **OF INDIVIDUALS.**

II. **SOME PROPERTIES HAVE A** HIGHER DEGREE OF INV **THAN OTHERS.**

III. **THE HIGHER THE DEGREE OF INVARIANCE OF A GIVEN PROPERTY, THE** GREATER **THE DEGREES OF** GENERALITY **&** NECESSITY **OF ITS LAWS/PRINCIPLES.**

I.e., DI(P1)>DI(P2) ➔ DN(Principles(P1))>DN(Principles(P2))

IV. **THE** HIGHER **THE DEGREE OF INVARIANCE OF A GIVEN** FIELD OF KN**, THE GREATER THE DEGREES OF** GEN & NEC **AVAILABLE TO ITS** LAWS/PRINCIPLES.

Significance:

Given that:

I. **EVERY PROPERTY IS** INVARIANT **UNDER SOME 1-1 AND ONTO REPLACEMENTS OF INDIVIDUALS,** r**,**

II. **SOME PROPERTIES, Including Some Properties Recognized by Humeans, HAVE A Fairly** HIGH DEGREE OF INV,

III. **There is a Connection Bet_** HIGH DEGREE OF INVARIANCE, NECESSITY**, and** LAWS, **and**

IV. **None of (I)-(III) are MYSTERIOUS,**

IT IS NEITHER SURPRISING NOR MYSTERIOUS THAT THERE ARE NECESSARY LAWS OF NATURE (Namely, LAWS GOVERNING/DESCRIBING-THE-BEHAVIOR OF HIGHLY INVARIANT NATURAL PROPERTIES).

CLAIM I.

EVERY PROPERTY IS INVARIANT UNDER SOME 1-1 [& Onto] REPLACEMENT OF INDIVIDUALS, r.

Notation:
D (Domain): a non-empty set of individuals.
r (Replacement Function): any 1-1 function on some D.

The claim: (∀P)(∃r) P is INV under r.

This is trivial, since every property is invariant under the IDENTITY REPLACEMENTS: functions r that replace each individual in D by itself.

But my claim is stronger:
PROPERTIES IN GENERAL ARE INVARIANT UNDER MORE THAN JUST IDENTITY r's.

CLAIM 2:

SOME PROPERTIES HAVE A HIGHER DEGREE OF INV THAN OTHERS:

Some properties P1, P2 are s.t. DI(P1) > DI(P2).

Ex.:

DI(*is-identical-to*) > DI(*partakes-in-gravity*).

Intuitively:

Partakes-in-gravity **is INV under r's of physical individuals by physical individuals but NOT under replacements of physical individuals by numbers.**
But *is-identical-to* is INV under both.

CLAIM 3:

THE HIGHER THE DEGREE OF INVARIANCE OF A GIVEN PROPERTY,
THE GREATER THE DEGREES OF GENERALITY & NECESSITY
OF ITS LAWS/PRINCIPLES.

I.e., DI(P1)>DI(P2) ➔ DN(Principles(P1))>DN(Principles(P2))

We saw that: **DI(*is-identical-to*) > DI(*partakes-in-gravity*).**

In fact: ***is-identical-to*** **is INV under all r's. It has a Max Degree of INV.**

To see that, take any r and any pair of individuals, <a,b>.

The image of <a,b> under r is a pair <c,d>.

Because r is 1-1: a=b iff c=d. I.e., *identity* is invariant under r.

Cont.

Now, take any Principle Governing *identity.* E.g.,

(ID) Every individual is identical to itself.

Claim: Since the property *is-identical-to* Does Not Distinguish bet_ ANY Individuals, the ID principle which governs it cannot Distinguish bet_ ANY Individuals.

THIS IS THE KEY POINT:

IF A PROPERTY P DOES NOT DISTINGUISH BETWEEN CERTAIN INDIVIDUALS, ITS PRINCIPLES CANNOT DISTINGUISH BET_ THEM EITHER,

WHERE BY "THE PRINCIPLES OF A PROPERTY P" I MEAN THE PRINCIPLES THAT DESCRIBE ITS BEHAVIOR OVER ALL OBJECTS, ACTUAL & COUNTERFACTUAL, TO WHICH IT APPLIES.

➔: THE MORE INDIVIDUALS A PROPERTY DOES NOT DISTINGUISH BET_ , THE GREATER THE SCOPE OF ITS PRINCIPLES.

Cont.

I.E., THE MORE 1-1 AND ONTO REPLACEMENTS OF INDIVIDUALS A PROPERTY P IS INV UNDER, THE GREATER THE ACTUAL-COUNTERFACTUAL SCOPE OF ITS PRINCIPLES.

Back to the Id property and its principle ID (Every individual is identical to itself):

Let us say that a principle is GENERAL if it has a LARGE ACTUAL scope,
And it is NECESSARY if it has both a LARGE ACTUAL scope and a large COUNTERFACTUAL scope.

➔ **(1) Since the principles of Id apply to ALL actual individuals, they are MAX_ General.**
(2) Since they apply to ALL counterfactual inds, they are MAX_ Necessary.

Cont.

In contrast: *Partakes-in-gravity* is NOT MAX_ Invariant.

➔ The principles of *gravity* are NOT MAX_ Gen or MAX_ Nec.

I.e: DN(Principles/Laws of *identity*) > DN(Principles/Laws of *gravity*).

And similarly for all other Properties and their Laws/Principles.

CLAIM 4:

THE HIGHER THE DEGREE OF INVARIANCE OF A GIVEN FIELD of KN (Discipline),
THE GREATER THE DEGREES OF ITS GENERALITY & NECESSITY

I.e., DI(F1)>DI(F2) ➔ DN(F1)>DN(F2).

Explanation:

Let us associate with each field of KN a characteristic property:

LOGIC: ***is-an-individual(-simpliciter)*****.**
PHYSICS: ***is-a-physical-individual*****.**
BIOLOGY: ***is-a-biological-individual*****.**

And let: DI(F) = DI(characteristic property, P, of F),
DN(F) = the degree of NEC of the principles/laws of P.

Then: Claim 4 follows from Claim 3.

Cont.

IT IS EASY TO SEE THAT:

DI(LOGIC)>DI(PHYSICS).

Hence:

DN(LOGIC)>DN(PHYSICS).

However, DI(physics) is still FAIRLY HIGH –

physics is INV under ALL r's of physical Individuals by physical Individuals.

Hence: DN(physics) is FAIRLY HIGH.

Cont.

Thus, take: ***partakes-in-gravity.***

IT IS INVARIANT UNDER THE SAME r's AS *is-a-physical-individual.*

I.E., *DI(partakes-in-gravity)*=DI(*is-a-physical-individual*).

➔ **DN(principles of gravity) is FAIRLY HIGH.**

➔ **THE PRINCIPLES OF GRAVITY – the principles that govern/describe-the-behavior of *gravity* over all individuals, actual and counterfactual, to which it applies – HAVE A SUFFICIENTLY HIGH DN TO COUNT AS LAWS.**

And there is nothing surprising or mysterious about that.

Definitions

Let r: D1→ D2 **be a 1-1 and onto function (bijection).**
Let P, P1, P2 **be properties.**

1. **Def of INV(**P,r**):**

   A. P **is 1-place 1st-level:**
   INV(P,r**) iff (∀x∈D1)[**P(**x**) ↔** P(r(**x**))].

   B. P **is 1-place 2nd-level:**
   INV(P,r**) iff (∀**P_{D1}**)[**P(**P_{D1}**)↔** P(r*(**P_{D1}**)],**

   where: (i) P_{D1} is a 1st-level property restricted to D1.
   (ii) r*(P_{D1}) is the image of P_{D1} under r.

2. Def of P is MAX-INV: (∀r)**INV(**P,r**)**

Cont.

3. Def of DI(P1)>DI(P2):

Case 1: At least one of P1, P2 is MAX-INV:

DI(P1)>DI(P2) iff {r: INV(P1,r)} ⊃ {r: INV(P2,r)}.

Case 2: Neither P1 nor P2 is MAX-INV:

Same, but r is restricted to domains with individuals that have at least one of P1, P2 if P1, P2 are 1st-level properties

(P_{D1}, P'_{D2} such that P1(P_{D1}), P2(P'_{D2}), if P1, P2 are 2nd-level properties).

Results for Logic, Math, Science

1. **All logical properties are MAX INV. ➔ All logical laws (consequences, truths) have a very high DN.**

2. **MAX INV captures the idea of FORMALITY in the objectual sense (high-structurality).**

3. **All higher-level math_ properties are FORMAL. ➔ All higher-order math_ laws have a very high DN.**

4. **Given that ALL 1st-order math_ individuals and properties are correlated with FORMAL properties (1 – ONE, 2 – TWO, … *is-even* – EVEN, …):**

   1st-order math can be viewed as representing FORMAL (higher-level) math.

   ➔ 1st-order math derives its strong NECESSITY from that of FORMAL (higher -level) math.

Results for METAPHYSICS & SCIENCE

1. **Necessity comes in Degrees. ➔ Possibility comes in Degrees – There are larger and smaller "spaces" of possibility.**

2. **This enables us to explain the difference bet_ Accidental & NEC Generality in Physics. Exs:**

(a) There are no very large (e.g. 1 mile in diameter) spheres of Gold on Earth.
(b) ... ... Uranium on Earth.

Fact: (a) is due to the fact that there is not much gold on Earth.
(b) is due to the fact the structure of Uranium rules out large Uranium spheres.

Now: Within the space of L_ possibilities, both are accidental.

But within the space of PHYSICAL possibilities,

(a) is accidental, (b) is NEC_:
(a) Is physically ACCIDENTAL; (b) is physically NEC_.

Result for MATH & SCIENCE

The view that Math_ properties are MAX INV (FORMAL) leads to a new answer to Wigner's Question:

HOW IS MATH APPLICABLE TO PHYSICS?

Our A: FORMAL properties, being MAX INV, don't distinguish bet_ individuals of any kind.

➔ They apply to all individuals, including PHYSICAL individuals.

➔ Math_ laws apply to PHYSICAL individuals..

There is nothing surprising or special about that.

Result for PHIL & SCIENCE

We have already seen how INV Th dissolves Humean worries about NEC_ LAWS.

(If there is INV in NATURE, there is a GROUNDWORK or INFRASTRUCTURE for NEC_ LAWS in NATURE.)

Another WORRY concerning LAWS of NATURE centers on their ABSTRACTNESS.

Humeans:

- **To abstract is to abstract from what is real. To falsify. To ignore what is real.**
- **Abstraction resides in us, not in Nature. Nature includes only particular objects and their particular properties.**

INV Th: NO.

Cont.

INV Th:

The real properties of real objects are SELECTIVE in character.

They themselves ignore, i.e., abstract from, various feature of (differences between) objects.

➔ Real Properties are already ABSTRACT in NATURE.

Some of these properties are abstract to a considerable degree.

These are the HIGHLY-INV Physical Properties (such as *gravity*).

SINCE INV, HENCE ABSTRCTION, IS BUILT INTO NATURE, NECESSARY LAWS ARE BUILT INTO IT AS WELL.

Cont.

REALISM:

Psillos (2014): The reality of laws is based on the reality of patterns and regularities.

INVARIANCE Th:

- **MAX INV is a mark of FORMAL patterns.**
- **PHYSICAL INV is a mark of PHYSICAL (or Physical & Math_) patterns.**

If, and to the extent that, there are objects and properties in nature, there is a network of INVARIANCES in nature, and these are the basis (one of the bases) for the reality of NATURAL PATTERNS, REGULARITIES, and LAWS.

LIMITS of INV Th

It is important to be clear about what INV Th purports to do and what it does not purport to do.

What it purports to do:

Provide a down-to-earth, non-mysterious, yet systematic, basis for the possibility of Nec laws in various fields: Physics, Math, L, etc.

In the case of MAX INV fields, such as L & Math, it actually establishes presence of Nec_ LAWS in them.

In the case of Non-MAX-INV fields, such as Physics & Biology, it establishes a GROUNDWORK or an INFRASTRUCTURE for NEC_ laws of various degrees of NEC.

It shows that there is a theoretical basis – a nonmysterious theoretical basis – for the existence of such laws.

Cont.

What INV Th does not purport to do:

1. INV Th does not purport to rule out additional Phil_ guidelines for, and even requirements on, laws and laws-supporting properties.
   Exs: Goodman – Projectibility;
   David Lewis – a balance bet_ naturalness & strength.

2. INV Th does not purport to determine what the laws of nature are or, even, whether there are laws of nature. This is left for scientists to do, partly on pragmatic grounds, but primarily by probing the world.

Cont.

3. The INV_ infrastructure of laws of nature is limited to laws that describe the behavior of HIGHLY-INV properties over all objects to which they in principle apply.

It does apply to laws of other kinds, e.g., stipulative and singular laws (e.g., laws concerning the speed of light, unless this is needed for highly-INV_ laws).

Such laws require either a Non-INV Expl or an extension of the INV_ Expl.

THANK YOU!

Exs. of INV

Case 1: 1st-level Properties.

Ex: is-a-human.

Consider:

D = {Obama, Tarski, 1, 2}.

r s.t.:

$\mathbf{r}(\text{Obama}) = \text{Clinton}$ $\qquad$ $\mathbf{r}(1) = 4$

$\mathbf{r}(\text{Tarski}) = \text{Frege}$ $\qquad$ $\mathbf{r}(2) = \text{Mt. Everest.}$

Claim: The 1st-level property *is-a-human* is invariant under **r**.

Cont.

$\mathbf{r}$(Obama) = Clinton
$\mathbf{r}$(Tarski) = Frege

$\mathbf{r}(1) = 4$
$\mathbf{r}(2)$ = Mt. Everest.

Claim: the 1st-level property *is-a-human* is invariant under $\mathbf{r}$.

Why?

b/c:

$\mathbf{r}$ replaces each individual which HAS the property *is-a-human*
by an individual which also HAS the property *is-a-human*,
&
each individual which DOESN'T HAVE the property *is-a-human*
by an individual which DOESN'T HAVE the property *is-a-human*.

Cont.

Is-a-human **is NOT invariant under:**

r': r'(Obama) =1 , r'(Tarski) = 2, r'(1) = Tree 1, r'(2) = Tree 2.

Cont.

Case 2: 2nd-level properties.

Ex: IS-A-PROPERTY-OF-MAMMALS

[This is a property of all 1st-level properties that are applicable in principle to some mammals].

Among the 1st-level properties that HAVE this 2nd-level property is the property *is-a-human*.

Among the 1st-level properties that DON'T HAVE this 2nd-level property is the property *is-a-number*.

Cont.

Case 2: 2nd-level properties.

Ex: IS-A-PROPERTY-OF-MAMMALS

Consider the domain D as above. I.e., D = {Obama, Tarski, 1, 2}.

Let r be a 1-1 function on D where:

r(Obama) = Dog 1 r(Tarski) = Dog 2, r(1) = Tree 1, r(2) = Tree 2.

Claim: IS-A-PROPERTY-OF-MAMMALS IS INVARIANT UNDER r.

Cont.

r(Obama) = Dog 1 r(Tarski) = Dog 2, r(1) = Tree 1, r(2) = Tree 2.

Claim: IS-A-PROPERTY-OF-MAMMALS IS INVARIANT UNDER r.

Why?

r INDUCES a replacement of the 1st-level property *is-a-human* by the 1st-level property *is-a-dog*.

But the 2nd-level property IS-A-PROPERTY-OF-MAMMALS does not notice this change.

From the point of view of IS-A-PROPERTY-OF-MAMMALS there is no difference between the 1st-level properties *is-a-human* and *is-a-dog*.

Cont.

IS-A-PROPERTY-OF-MAMMALS is NOT invariant under

r':
r'(Obama) = Tree 1, r'(Tarski) = Tree 2, r'(1) = Snake 1, r'(2) = Snake 2.

r' induces a replacement of *is-a-human* by *is-a-tree.*
and *is-a-tree* is not A-PROPERTY-OF-MAMMALS.

IS-A-NONEMPTY-PROPERTY is INV under ALL r's.

Take any D and **r** on D.
Now, take any 1st-level 1-place property P.

2 cases:
1. P is NONEMPTY in D.
2. P is EMPTY in D.

Now consider the image of D and P under **r** in both cases.
Let's call the image of D under **r** D'.

1. **r**(P) is NONEMPTY in D'.
2. **r**(P) is EMPTY in D'.

Result: IS-A-NONEMPTY-PROPERTY is INV under **r**.

Since D, **r**, P are arbitrary:

IS-A-NONEMPTY-PROPERTY is INV under ALL **r**'s.