Model of opinion dynamics in a social network when opinions are in logical relations

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The paper proposes a new model for changing the agent's opinions, taking into account the damage that may cause him contradictions between the opinions he holds. Such a model allows us to more finely investigate the effect of agents choosing their opinions when they are inconsistent. In social networks, this is a frequent case, for example, often such contradictions can be found in the speeches of politicians, however, many voters nevertheless value precisely the totality of opinions without internal contradictions.

Let us consider a set of agents $N = \{1, \ldots, n\}$ and a set of formulas $\Phi = \{\varphi_1, \ldots, \varphi_{2m}\}$, such that for any $i \leq m$ $\varphi_i = \neg \varphi_{i+m}$, where $\neg$ is a negation. Agents opinion at a moment t we describe by a matrix $X = \{x_{ij}^0 \in [0;1]\}$, where x_{ij} is how much an agent i believes that φ_j is true. At each step t, the agent can choose which statements to support. Agent i maximizes criteria F_i, that is described by a sum of some functions : degree of inconsistency (another approach is described I [1])

$$c_i^t\left(\left\{x_{ij}^t\right\}, L = \left\{L_{ij}\right\}_{i\in[1,\ldots,l],\, j\in[1,\ldots,2m]}\right) = \sum_{k\in[1,\ldots,l]} \begin{cases} d_{ik}, \sum\limits_{j\in M} L_{kj} x_{ij}^t = \sum\limits_{j\in M} L_{kj} \\ 0, \sum\limits_{j\in M} L_{kj} x_{ij}^t \neq \sum\limits_{j\in M} L_{kj} \end{cases}, d_{ik} > 0 \tag{1}$$

that depends on the statements which she chooses and the matrix L describing their consistency, inconsistencies with their own initial support

$$s_i^{in} = b_i \sum_{j\in M} \left(x_{ij}^0 - x_{ij}^t\right)^2 \tag{2}$$

and inconsistencies with the opinions of her friends

$$s_i^{in} = \left(1 - b_i\right) \sum_{k\in N} \sum_{j\in M} a_{ijk} \left(x_{ij}^t - x_{kj}^t\right)^2 \tag{3}$$

For simplicity, we assume that the criterion is the sum of the values of functions with weights $r_i^0, r_i^{in}, r_i^{ex}$,

$$F_i(t) = -\left(r_i^0 c_i^t + r_i^{in} s_i^{in}(t) + r_i^{ex} s_i^{ex}(t)\right) \tag{4}$$

Weights $r = \left(r_i^0, r_i^{in}, r_i^{ex}\right)$ will be called as a type of an agent i.

Parameters $\{a_{ij}\}$ describe the relationship between agents, $a_{ij} = a$ means that agent i will lose if his attitude to opinion k is opposite to opinion of agent j. The b_j parameter shows how much agent j loses if his public opinion is the opposite of his inner conviction. The d_{ik} parameter shows how unpleasant for agent i the contradiction is the k-th contradiction in his beliefs. We will examine and compare heuristics that real agents can adhere to. The task of describing opinions and exchanging opinions is important [2, 3] and attracts the attention of researchers. The closest analogues are: the French-deGroot model. In this model, inconsistency with the support of friends is considered, but two other functions are not considered [4,5], the Friedkin – Johnsen model. This model considers inconsistency with the support of friends and inconsistency with their own initial support, does not consider inconsistency [6], the Fridkin-Proskurnikov-Parsegov-Tempo model, where both inconsistencies are considered, and the relationship of arguments, but this relationship is modeled in a different way [7]. So most of the novelty of the model is connected with inconsistency, we give examples. More detailed information on the logic and actions of agents is, for example, here [8].

Example 1. Let the statements $\Phi_1 = \{p, q, \neg p, \neg q\}$, i.e. two independent statements and their two negations. The matrix L is as follows (under the assumption that the statements $x = p \wedge \neg p$ and $y = q \wedge \neg q$ are false)

$$L_1 = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \tag{5}$$

Example 2. Let statements $\Phi_2 = \{\perp, p, p \to q, q, T, \neg p, \neg(p \to q), \neg q\}$, those. two independent statements and their two negations, where $\perp$ is false, T is a truth. Matrix L could be the following

$$L_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \qquad (6)$$

where

$$\frac{p, p \to q}{q}, \quad p \to q = \neg p \vee q, \quad \neg(p \to q) = \neg(\neg p \vee q) = p \wedge \neg q \qquad (7)$$

Matrix L does not have to describe all possible contradictions - in it it can describe only part of them, since it describes the subjective opinions of agents regarding what is contradictory and what is not. So one agent can consider possessing logical abilities and clearly see contradictions, and the other such abilities not possess and not feel discomfort with almost obvious logical contradictions. The extreme pragmatism of agents can lead to the same effect.

An algorithm for implementing code in Python or another programming language: (1) Write down the logical constraint in the form of the matrix L, set the types of agents, write out the initial beliefs of the agents, form a network with weights, specify aij, set the number of experiment steps. (2) Start the iteration cycle the best answers for each agent - choosing his new beliefs based on maximizing his criterion according to his type. (3) If the beliefs have not stabilized (are not equal to the previous ones) and the number of steps has not been exceeded, go to step 2, assign the updated beliefs of the agents in accordance with the results of the search carried out in paragraph 2. (4) Display statistics.

An interesting extension of the model can be the model of evolutionary games, when a series of games is considered and in each new game the type of agent shifts to the type of agents that received the greatest gain in the previous game. Assumption: depending on the initial conditions in a stable state, agents with a low dependence of the criterion on the degree of inconsistency can dominate quantitatively, and vice versa.

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