Talk Outline

- Introduction: Kinds of Modalities
- Logical Square and Its Generalizations
- Interpretation of Diagrams and Philosophical Issues

Introduction: Kinds of Modalities

Alethic (necessity, etc, truth, etc.)

A historical remark: Kant contrasted assertotic, necessary and problematic sentences; he did not regard assertoric as modal; one can guess that this mistaken view blocked the development of modal logic;

Deontic (obligatory, etc.)

Cont.

Epistemic (known, guessed, certain, etc.)

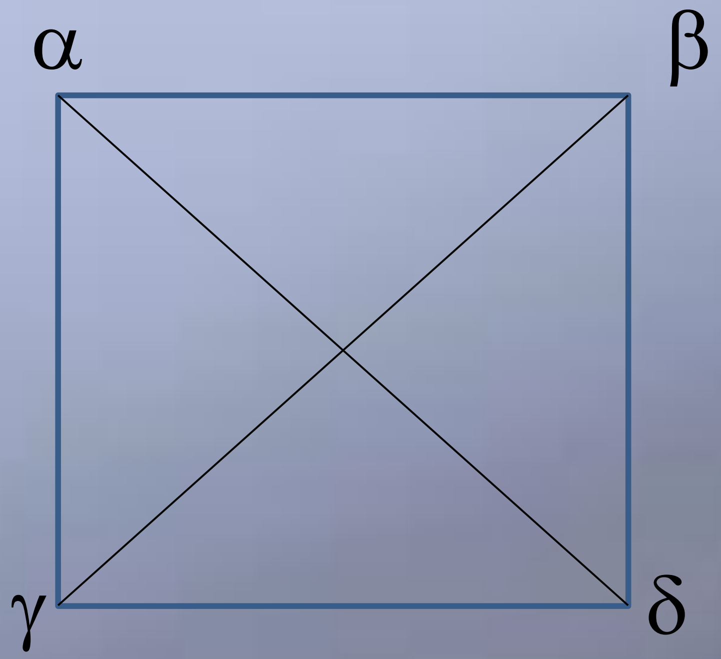
Erotetic (ask, etc.) ….

A general form: Mod(*A*), sometimes with with individual parameter (*a* performs that *A*) sometimes without (it is necessary that *A*)

Remark: de dicto, de re (predications) ; I will treat them as equivalent, it is true that *A*, *A* is true *;*

Symbolic: ■*A* ◆ A

Logical Square



CONT

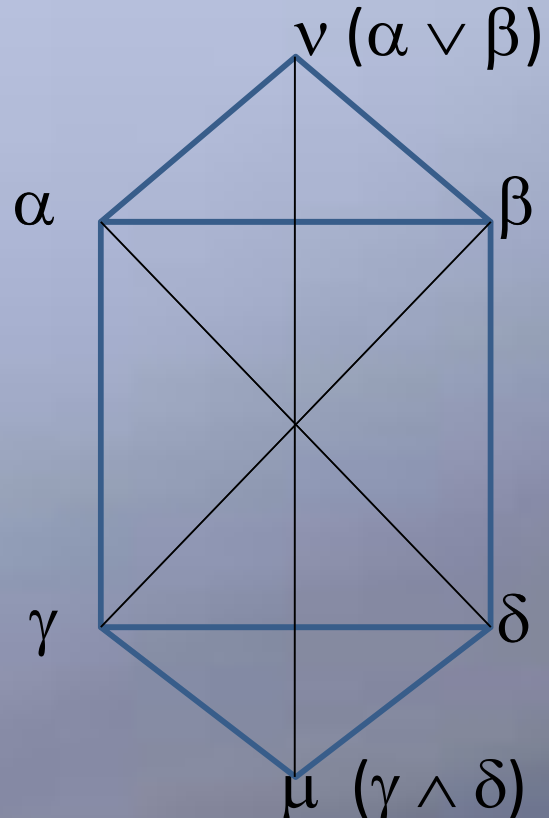
α-■*A*, β-■$\neg$*A*, γ - $\neg$■$\neg$*A*, ◆δ - $\neg$■*A*, ◆$\neg$*A*

(a) $\alpha \Rightarrow \gamma$,(b) $\beta \Rightarrow \delta$ (c) $\neg(\alpha \wedge \beta)$; (d) $\gamma \vee \delta$

(e) $\alpha \Leftrightarrow \neg\delta$; (f) $\beta \Leftrightarrow \neg\gamma$.

Readings of (a) necessity implies possibility, obligation implies permission, *a* is good implies it is not so that *a* is bad (it is good that *A* implies that that it it is not so that $\neg$*A* is bad, *A* is true implies that $\neg$ *A* is not truer (false?), etc. Observe that γ has no ordinary wording in some cases.

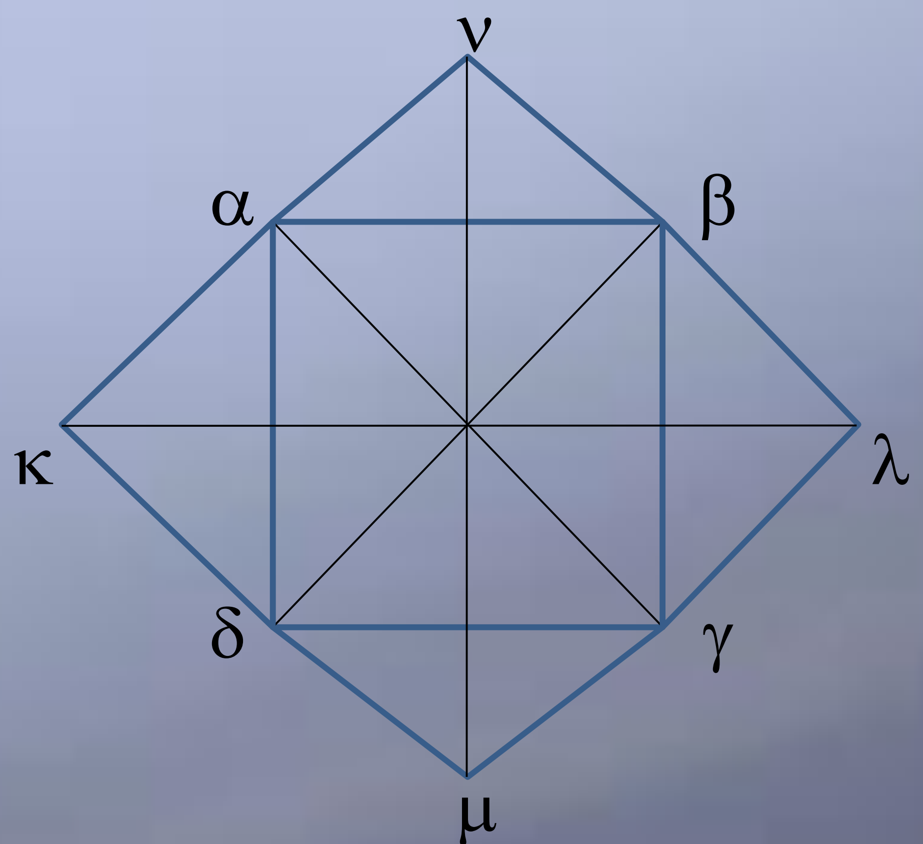
LOGICAL OCTAGON



New rules: $\alpha \Rightarrow \nu$, $\mu \Rightarrow \gamma$

LOGICAL OCTAGON

New $\kappa - A$, $\lambda - \neg A$



Some Conceptual Issues

What is contingency (permission, δ or μ)?

The rule $\blacksquare A \Rightarrow A$ is valid for necessity and truth, but not for obligation and knowledge – hence the classical concept of knowledge (true justified belief is not grounded in logic).

If we adopt $A \Rightarrow \blacksquare A$, T-scheme obtains. It reduces $\neg TA$ to $T\neg A$ (not-true to true not, let say false). However, bivalence, that is, generalized ν for truth is not a logical truth, contrary to $\alpha \vee \delta$ (*A* is true or A not true). Consequently, the logic of diagrams does not excludes many-valueness or truth-value gaps. This confirms Łukasiewicz's view that the principle of bivalence is metalogical and adopted on extralogical grounds. A remark on Leśniewski, his protothetic has the principle of bivalence as a logical theorem but it assume extensionality and T-scheme. Truth is lacking the modal status. Incidentally, since the formula $A \Rightarrow \blacksquare A$ is responsible for paradoxes, we have some material to explain how pure logic is involved into semantic troubles.

Negative Theory of *Malum*

Briefly speaking, the negative theory of *malum* (**NTM)**, shared by the followers of Thomas Aquinas, considers wrongness as *negativum* or rather *privativum.* This means that *malum* (evil) is the lack of goodness. **NTM** is a consequence of the medieval ontologico-semantic theory of transcendentals (*transcendentalia*), that is, ovecategorial concepts; according to a popular statement *transcendentalia omnia genera transcendunt*. *Ens* (being), *verum* (truth) and *bonum* (goodness) are typical examples of transcendentals. The main thesis concerning overcategorial concepts asserts that they are co-extensional and mutually convertible. This general principle has various particular instances, expressed by Latin statements, like *ens et verum convertuntur*; *ens et bonum convertuntur* and *verum et bonum convertuntur.* **NTM** is connected with the second, that is, being and godness are convertible. Its application to the problem of evil is immediate. Clearly, *malum* opposes *bonum*. Hence, if being and goodness are convertible, *malum* is negative and, according to **NTM**, does not exist, because it is not a being at all. As I already noted, *malum* is rather a *privativum* with respect to *bonum*, but not its *negativum.* Disregarding various points which differ *negativa* and *privativa*, in particular, metaphysical issues, let me only remark that, logically speaking, if **X** is a *positivum* and **Y** constitutes its *negativum*, both are contradictories. On the other hand, related *positiva* and *privativa* are contraries. Now, the fundamental claim of **NTM** states that *malum* and *bonum* are contraries as *privativa.* This statement as well as the convertibility thesis are the basic core of **NTM**.

Cont

- The convertibility of *ens* and *bonum* can be précised as follows:
- (1) *a* is an *ens* if and only if *a* is a *bonum*.
- (1) can be split into two parts, namely (2) if *a* is a *bonum*, then *a* is an *ens*; (3) if *a* is an *ens*, then *a* is a *bonum*; - necessary in **NTM**.
- (2) is rather trivial as an empirical assertion, because if an existing object is good, it must be something. The real problem concerns (3). Consider sentences about *bonum* and *malum* as modals. More specifically, we interpret α as 'it is good that *A*', β as 'it is wrong that *A*', γ as 'it is not wrong that *A*', δ as 'it is not good that *A*', ν as 'it is good or wrong that *A*', μ as 'it is neither good nor wrong that *A*', κ as 'it obtains that *A*', and λ as 'it does not obtain that *A*'. Thus, (a) *bonum* and *malum* are contraries (as desired by **NTM**); (b) if something is good, it is not wrong (plausible); (c) if something is wrong, it is not good (plausible); (d) *bonum* and *non-bonum* are contradictories (plausible); (e) *malum* and *non-malum* are contradictories (plausible). Thus, logical truths based on **OCTAGON** are fully coherent with our axiological intuitions; in fact, **SQUARE** is enough here.

Cont

- However, **NTM** makes further claims. They are summarized in
- (4) every object is good; formally: $\forall x\alpha(x)$;
- (5) every object is wrong; formally: $\forall x\beta(x)$;
- (6) every object is good or wrong; formally: $\forall x(\alpha(x) \vee \beta(x))$, $\forall x\nu(x)$;
- (7) every object is indifferent; formally: $\forall x(\neg\alpha(x) \wedge\neg\beta(x))$, $\forall x\mu(x)$;
- (8) some objects are good, some wrong and some indifferent; formally:
- $\exists x\alpha(x) \wedge \exists x\beta(x) \wedge \exists x\mu(x)$.
- The formulas (4)-(8) generate various possibilities of distribution of *bonum* and *malum* over *ens* understood as the universal collection of existing objects.

Cont

- We can baptize particular eventualities in a way (some labels are ad hoc):
- (A) radical ontological ethism (ontological pantethism) with three special instances:
- (a) monism of *bonum* – (4);
- (b) monism of *malum* – (5);
- (c) dualism of *bonum* and *malum* – (6);
- (B) moderate ontological ethism – (7);
- (C) ethical ontological indifferentism – (8).
- Ontological panethism says that every existing object is ethically positively valuable, that is good. More specifically, (Aa) asserts that only goodness can exist (this is simply a version of **NTE**), (Ab) maintains that only evil and nothing more is actual exist (Schopenhauer's position or radical ethical pessimism), and (Ac) proposes that both goodness and evil can exist (manicheism of *bonum* and *malum*). Moderate ontological ethism (see (B)) says that there are valuable (good or wrong) as well as indifferent objects; this view seems to be closely related to the ordinary account concerning the distribution ethical values over the world in which we act. Finally, ethical ontological indifferentism (see (C)) considers being as ethically not determined (indifferent), that is, neither good nor wrong.

Cont

- All views depicted by (4) – (8) are logically possible, but **NTM** takes one of them as necessary and excludes other, including that qualified above as the most popular, as erroneous, not only empirically, but also conceptually. Although the sense of necessity as used by the defenders of **NTM** is vague, this theory has no justification in logic. Another defect of **NTM** consists in passing from being to goodness. Formally speaking, (21) says (the symbol ■ means 'it is good that' in the present context)
- (9) $A \Leftrightarrow \blacksquare A$.
- However, neither (2) nor (3), and, *a fortiori*, (9) as well have justification in logic. If (D3) is applied to axiology, we have no reason to maintain that resulting logic is normal, that is, validates the formulas $\alpha \Rightarrow \kappa$ and $\kappa \Rightarrow \alpha$. Thus, **NTM** is committed to the naturalistic fallacy. Observe that the above analysis does not appeal to subjectivism, objectivism, emotivism, cognitivism, etc. I agree if one will say that my analysis in this section is purely negative and consists in giving reasons for rejecting **NTM**. However, it was not my task to develop a positive theory of *bonum* and *malum*, but I only intended to show how our simple logic of **OCTAGON** helps in testing philosophical proposals.
- A similar argument shows that theses on the convertibility of *ens* and *verum* or *v*erum and *bonum* are problematic.

The Hume Thesis

- Hume observed in his *Treatise on Human Nature* that is does not entail ought. More precisely, the Hume thesis (**HT**) asserts that sentences of the form '*a* is *b*' do not logically entail sentences of the form '*a* ought to be *b*'. **HT** finds its full logical justification in the logic generated by **OCTAGON**. Is-sentences are located at the point κ, but ought-sentences occupy the place α. Since the sentence $A \Rightarrow \blacksquare A$ is not validated,, **HT** holds. However, there is much more to say. First of all, since a part of the standard deontic logic belongs to our logic of (**D3**), its non-normality entails that the converse of **HT**, that is, $\neg \vdash(\alpha \Rightarrow \kappa)$ holds as well. Secondly, we have the simple and converse **HT** for permissions and indifferences, because the formulas $A \Rightarrow \blacklozenge A$, $\blacklozenge A \Rightarrow A$, $A \Rightarrow \blacklozenge{}^{**}A$, $A \blacklozenge{}^{**} \Rightarrow \blacklozenge{}^{**} A$ ($\blacklozenge{}^{**}$ refers to indifference) are not logically truth. In general, if $\blacktriangledown$ expresses any normative (deontic) operator, the implications $A \Rightarrow \blacktriangledown A$ and $\blacktriangledown A \Rightarrow A$ are not logical principles. A remarkable fact is that **HT** as it is conceived here, makes no reference to norms as specific sentences which are neither true nor false. Ought-sentences are normal declarative sentences. Thus, in order to justify **HT**, we do not need to appeal to metaethical positions mentioned at the end of previous section. I claim that **HT** applies to many other modalities, including epistemic. Look once again at the relation between *A* and to know that *A.*

Determinism

- Now, let the form $\blacksquare A$ means 'A is determined'. Accordingly, we interpret $\blacksquare\neg A$ as '$\neg A$ is determined' (A is antidetermined), $\blacklozenge A$ as 'it is not true that $\neg A$ is determined' (note the symbol $\blacklozenge$ has no simple wording), $\blacklozenge A$ as 'A is not determined' (or A is contingent[*]; this reading is admissible in the case of determinism) and ©[**] A as 'A is not determined and $\neg A$ is not determined' (or A is contingent[**]). To formulate claims related to various forms determinism and indeterminism, we need following quantified closures:
- (10) every A is determined; formally: $\forall A \blacksquare A$;
- (11) every $\neg A$ is determined, every A is antidetermined; formally: $\forall A \blacksquare \neg A$;
- (12) every A is determined or antidetermined; formally: $\forall A(\blacksquare A \vee \blacksquare \neg A)$;
- (13) every A is contingent[**]; formally: $\forall A(\blacklozenge A \wedge \blacklozenge \neg A)$, $\forall A$ ©[**] A;
- (14) some A are determined, some A are antidetermined, some A are contingent[**];
- formally: $\exists A \blacksquare A \wedge \exists A \blacksquare \neg A \wedge \blacklozenge A$.
- The theses (10) – (12) can be considered as formulations of radical determinism (everything is determined). (13) expresses radical indeterminism (nothing is determined, everything is contingent[**], but (14) (something is determined, something is contingent) can be accepted by moderate determinists and moderate indeterminists.

Cont

- This analysis of determinism is exactly twin with respect to that concerning *bonum* and *malum*, although we have here no counterpart of **NTM**; thus, similarity is formal, not substantial. All formulas in the sequence (30) – (34) are not tautologies and thereby they exceed logic. Hence, purely logical criteria cannot decide, which view about the order of reality is correct and how, if in any way, the world is structured, for example, by causality or other mechanism of regulation. This fact strongly suggests that the thesis that classical logic, in particular, the principle of excluded middle entails radical determinism is very implausible. Although this note about determinism is very preparatory, it clearly shows that without an introductory analysis of the issue further steps hang in vain. For instance, the difference between moderate determinism and moderate indeterminism reduces itself to a distribution of determinacies and contingencies.
- Similar issues: complete normative systems (everything is obliged or prohibited – a dream of bureaucrats, anarchistic normative system – nothing is obligatory or prohibited, typical normative systems something is obligatory, something is prohibited, and something is permitted (indifferent), how to introduce normative qualifications (no set of indifferences constitutes a normative system)
- Omniscience

Consistency, etc.

- In this section modalities are applied to sets of sentences, not single utterances. Let Γ be a set of sentences. In order to eliminate some inessential complications, we assume that Γ, if consistent (the set of logical consequences of Γ is different from the set of all sentences). If Γ is a set of sentences, it's a denial of the set Γ', such that for some $A \in \Gamma$, $\neg A \in \Gamma'$. Thus, Γ' is a denial of Γ, if the former contains at least one negation of a sentence belonging to the latter. A set Γ is true, if all its elements are true, and false, if it contains at least one falsehood. Now, we interpret ■Γ as 'Γ is true', ◘Γ as 'Γ is inconsistent', ◆Γ as 'Γ is consistent' and ◆$\neg\Gamma$ as 'Γ is false'. Accordingly (to **SQUARE**) we have
- (15) truth implies consistency;
- (16) inconsistency implies falsehood;
- (17) truth and inconsistency are contraries;
- (18) consistency and falsehood are complementaries.
- (19) truth and falsehood are contradictories;
- (20) consistency and inconsistency are contradictories.
- The last dependency opens the possibility of set of sentence, which are consistent and false. In fact, if Γ is consistent, Γ' preserves this property as well.

Cont

- Assume that Γ axiomatizes a theory **T** enough for expressing Peano arithmetic; this means that **T** = *Cn*Γ. We say that **T** is ω-consistent, if **T** $\vdash$ *P1, P2, P3*,…, then $\neg$(**T** $\vdash$ $\exists n \neg Pn$). **T** is ω-inconsistent: if **T** $\vdash$ *P1, P2, P3*, …, and **T** $\vdash$ $\exists n \neg Pn$. At first, I consider relations between consistency, ω-consistency, inconsistency and ω-consistency without referring to truth and falsehood. This means that the considerations entirely remain within syntax. Interpret the particular points of **SQUARE** in the following way: α as '**T** is ω-consistent'; β as '**T** is inconsistent'; γ as '**T** is consistent'; and δ as '**T** is ω-inconsistent'. Thus, we have
- (21) ω-consistency entails consistency;
- (22) inconsistency entails ω-inconsistency;
- (23) ω-consistency and inconsistency are contraries;
- (24) consistency and ω-inconsistency are complementaries;
- (25) ω-consistency and ω-inconsistency are contradictories;
- (26) consistency and inconsistency are contradictories,
- The reason for (22) is that inconsistency entails everything. Theories which are consistent and ω-inconsistent are just a case in which the point .

Cont

- If **T** is consistent, does not matter whether ω-consistent or ω-inconsistent, it has a model, due to a general theorem that every consistent theory has a model. This fact leads us to semantics. However, there is a problem how to embed truth into our framework, because **T**, if ω-inconsistent is true in a model. Moreover, if **T** is ω-inconsistent, it also contains the axioms Γ; this is the main reason that we cannot limit our considerations to Γ. As it is well-known, ω-consistent arithmetic is true in the so-called standard model, but ω-inconsistent systems are true in non-standard models (they have universes with "non-standard" numbers, in particular, greater of any standard number). Thus, we cannot use truth (falsehood) *simpliciter*, but standard and non-standard truth. Interpret α as '**T** is standardly true'; β as '**T** inconsistent'; κ as '**T** is ω-consistent'; λ as '**T** is ω-inconsistent'; γ as '**T** is consistent'; and δ as '**T** is non-standardly true'.

Cont

- This justifies the following additional (with respect to (21 – 26) theorems:
- (27) standard truth and inconsistency are contraries;
- (28) standard truth entails ω-consistency;
- (29) ω-inconsistency entails non-standard truth;
- (30) non-standard truth entails ω-inconsistency;
- (31) ω-consistency and non-standard truth are complementiaries.
- (32) consistency and non-standard truth are complementaries.
- One can say that non-standard truths are standard falsehoods. Hence, a theory with at least non-standard truth is standardly false. (31), does not exclude theories which are standardly false and ω-consistent. (32) just implies that theories consistent and standardly false are possible. Moreover, it is clear now that the assumtion of consistency is not enough for proving Gödel' first theorem, because is does not exclude standardly false conclusions. Rosser's improvement can be regarded as blocking this possibility.

Cont

- The above considerations show the significance of the concept of truth in a model and the distinction between standard and non-standard models. Since truth in a model, standard as well as non-standard, depends on interpretation, we see that the concept of interpretation is crucial for semantics. Now the problem is whether the concept of standardness can be reasonably generalized beyond metamathematics. This problem is connected with questions, like, for example, “what were happen, if the sentence ‘snow is white’ would be understood as ‘grass is green’. The suggested answer is that nothing, but the change of the standard understanding of the words ‘snow’ and ‘white’ or ‘grass’ and ‘green’. Although we cannot apply the concepts of ω-consistency and ω-inconsistency outside arithmetic, the notion of the coherence with typical usages of language item seems a certain substitute. Since this remarks are very tentative, I do not try to formulate an exact definition, which perhaps is even impossible. However, the idea that standard truth implies coherence, incoherence (an analogon of ω-inconsistency) implies non-standard truth (standard falsehood), but possible are situations in which we have to do with consistent and simultaneously incoherent systems, seems philosophically attractive. However, one should always remember that everything depends on interpretations. Thus, the concept of standardness is not purely semantic, but it has pragmatic flavor.