Eugeny K. Voishvillo: Russian Plan for Relevant Logic

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E.K. Voishvillo 1913–2008

1934 - Metallurgical college

1939 - Faculty of Mechanics and Mathematics, Kazan University

1943-46 - World War II

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1939 - Faculty of Mechanics and Mathematics, Kazan University

1943-46 - World War II

1946 - Logic Training Courses

1949 - Candidate Dissertation

1967 - Doctoral Dissertation

1949–2008 - Dept of Logic, Faculty of Philosophy, Lomonosov Moscow University

Main Ideas

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("relevantization" of classical logic)

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(law of science, dispositional predicates, scientific explanation, counterfactuals)

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Information and Semantics

Ackermann's idea of *Strenge Implikation*:
A implays B means that "the content of B is part of the content of A"

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Voishvillo's answer: Logical content may interpreted via semantic conception of information.

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Bar-Hillel-Carnap on information and semantic content:
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Example:
Let the initial set of possibilities M for an arbitrary formula be the set of possible assignments to propositional variables in it, and consider $p \wedge q$.
Then M for $p \wedge q$ is four-element set and $M_{p \wedge q}$ is its singleton subset.

Information and Semantics

Voishvillo on information:
An information of an arbitrary statement A – $I(A, M)$ – is a pair $\langle M_A, M \rangle$,
M_A denotes a set of situations from initial set M, where A is true.

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Example:
Consider $p \wedge q \vDash p$.
It is evident that $M_{p \wedge q} \subseteq M_p$ and, thus Ip is part of $Ip \wedge q$.

Information and Semantics

Definition 1.

$A \vDash B \Leftrightarrow$ the logical content of B is part of the logical content of A

$$\Leftrightarrow I(B, M) \subseteq I(A, M)$$

$$\Leftrightarrow \langle M_A, M \rangle \text{ is part of } \langle M_B, M \rangle$$

$$\Leftrightarrow M_A \subseteq M_B.$$

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Definition 2.

$M_A = \{\alpha : \text{A is true in } \alpha\}$.

Information and Semantics

M is a set of state-descriptions – a class of atomic sentences in corresponding language $(\neg, \wedge, \vee)$, containing exactly one of A or $\neg A$ for each atomic sentence A.

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Let TA/α (FA/α) means "A is true (false) in state-description α".

Definition 3.

$Tp_i/\alpha \Leftrightarrow p_i \in \alpha; \quad Fp_i/\alpha \Leftrightarrow \neg p_i \in \alpha.$

$T\neg B/\alpha \Leftrightarrow FB/\alpha; \quad F\neg B/\alpha \Leftrightarrow TB/\alpha.$

$T(B \vee C)/\alpha \Leftrightarrow TB/\alpha$ or TC/α;
$F(B \vee C)/\alpha \Leftrightarrow FB/\alpha$ and FC/α .

$T(B \wedge C)/\alpha \Leftrightarrow TB/\alpha$ and TC/α;
$F(B \wedge C)/\alpha \Leftrightarrow FB/\alpha$ or FC/α.

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Thus,
- for tautology $(\top)$, $I(\top, M) = \langle M, M\rangle$, and so, any tautology has null informative content, and, hence, $\forall B, B \vDash \top$

- for contradiction $(\bot)$, $I(\bot, M) = \langle \varnothing, M\rangle$, and so, any contradiction contains the greatest amount of semantic information, and, hence, $\forall B, \bot \vDash B$

Information and Semantics

Voishvillo claim:

Standard (classical) state-description meets two conditions for arbitrary propositional variable p_i:

(a) $p_i \notin \alpha$ or $\neg p_i \notin \alpha$;

(b) $p_i \in \alpha$ or $\neg p_i \in \alpha$.

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Therefore, instead of considering $I(A)$ we consider $I(A/\Gamma)$, where $\Gamma = \{a, b\}$.

To avoid paradoxes, the conditions a and b should be abandoned!

Information and Semantics

Definition 4.

A set of generalized state-descriptions is an arbitrary sub-set of the set of literals.

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If we consider an extra condition

(c) $\exists p_i(p_i \in \alpha \text{ and } \neg p_i \in \alpha) \Rightarrow (q_j \in \alpha \text{ or } \neg q_j \in \alpha)$,

then in a natural way first-degree fragments of **R** (**E**), **Kl**, **LP** and **RM** are defined.

Deductive Principles

1. First-Degree principles
2. $A \longrightarrow A$
3. $(A \longrightarrow B) \longrightarrow (A \supset B)$
4. Principles of relevant weakening

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Formula A is informationally weakened if some of its positive subformulas are replaced by weaker formulas (according to some logical law of the type $C \longrightarrow D$) or if some of its negative subformulas are substituted with stronger formulas.

Example:

$(p \wedge q) \longrightarrow p$ and $p \longrightarrow (p \vee q)$ are weaker than $p \longrightarrow p$.

Weakening Principles

Some examples:

WP1 $\models A \longrightarrow (B \longrightarrow B^w_A)$,
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As a result, Voishvillo obtains system **E**.
There are some extra principles, which validate, for instance, Urquhart formula:
$((A \longrightarrow B) \wedge (C \longrightarrow (A \vee D))) \longrightarrow (C \longrightarrow (B \vee D))$.

Natural Deduction

$$\frac{A \longrightarrow B[\Gamma], A[\Delta]}{B[\Gamma, \Delta]} \quad [\longrightarrow \mathrm{E}] \quad \frac{B[\Gamma, A]}{A \longrightarrow B[\Gamma]} \quad [\longrightarrow \mathrm{I}],$$

where Γ and Δ are lists (sequences) of formulas.

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Structural rules:

Contraction $$\frac{A[\Gamma, B, B, \Delta]}{A[\Gamma, B, \Delta]}$$

Permutation $$\frac{A[\Gamma, B, C, \Delta]}{A[\Gamma, C, B, \Delta]}$$

or Restricted Permutation $$\frac{A[\Gamma, B, (C \longrightarrow D), \Delta]}{A[\Gamma, (C \longrightarrow D), B, \Delta]}.$$

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THANKS FOR YOUR TIME