The Concept of Knowledge in the Philosophical Epistemology
Homotopy Type Theory as KR framework
Concluding Remarks

Knowledge Representation with HoTT

Andrei Rodin (IPRAS/HSE), jww Serge Kovalyov (ICSRAS)

TYPES 2019, Oslo, June 11-14, 2019

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JTB

Knowledge as Justified True Belief:

Subject S knows that p (where p is a proposition) just in case the following three conditions are satisfied:

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JTB

Knowledge as Justified True Belief:

Subject S knows that p (where p is a proposition) just in case the following three conditions are satisfied:

1. p is true

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JTB

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Subject S knows that p (where p is a proposition) just in case the
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1. p is true
2. S believes that p

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JTB

Knowledge as Justified True Belief:

Subject S knows that p (where p is a proposition) just in case the following three conditions are satisfied:

1. p is true
2. S believes that p
3. S is justified in believing that p.

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Remark:

JTB identifies knowledge with knowledge of certain proposition or propositions.

This type of knowledge is conventionally referred to as propositional knowledge aka knowledge-that.

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Gettier 1963

$$\frac{16}{64} = \frac{1\!\!\!/6}{\!\!\!/64} = \frac{1}{4}$$

$$\frac{26}{65} = \frac{2\!\!\!/6}{\!\!\!/65} = \frac{2}{5}$$

No satisfactory formal account was available until recently, see below.

Constructive Logic

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Constructive Logic

- Truth as the existence of proof; the JTB-tripartition does not apply.

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Constructive Logic

- Truth as the existence of proof; the JTB-tripartition does not apply.
- Epistemically-laden conception of logic as the "determination of the best available evidence" (Cohen&Nagel 1934)

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Constructive Logic

- Truth as the existence of proof; the JTB-tripartition does not apply.
- Epistemically-laden conception of logic as the "determination of the best available evidence" (Cohen&Nagel 1934)
- Justification Logic (Artemov&Fitting 2019), Proof-Theoretic Semantics, MLTT/HoTT

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The constructive approach to knowledge gives justification *even more* importance. Thus justification is a key element of knowledge both under the JTB and the constructive conceptions of knowledge.

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KR conceptions of knowledge

Ex.: Knowledge is the "whole body of data and information that
people bring to bear to practical use in action" (after Jukus et al.
2013)

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Ex.: Knowledge is the "whole body of data and information that people bring to bear to practical use in action" (after Jukus et al. 2013)

No mention of justification in the studied CS literature!

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Ontology vs. Epistemology

Since late 1980-ies the concept of ontology has been widely used in KR as a formal semantic tool.

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Ontology vs. Epistemology

Since late 1980-ies the concept of ontology has been widely used in KR as a formal semantic tool.

In philosophy *ontology* is a discipline that accounts for being/existence.

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Epistemology accounts for knowledge.

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Ontology vs. Epistemology

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In philosophy *ontology* is a discipline that accounts for being/existence.

Epistemology accounts for knowledge.

Puzzle: Why KR makes use of (formal) ontology but not of (formal) epistemology?

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Justification as a practical issue

A regular user of KR system has no means to verify/justify an
information obtained via the system provided by this very system.
The existing verification technologies are not designed to be
available to regular users.

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Desiderata for KR:

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Desiderata for KR:

▶ Support justification/verification in form of routine procedure available to all users;

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Desiderata for KR:

- ▶ Support justification/verification in form of routine procedure available to all users;
- ▶ Support justification/verification in form, which is specific w.r.t. obtained information: specific evidences rather than general assurances.

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A Reason Why Justification is not Supported:

Standard logical tools and gadgets such as Classical first-order logic with the Tarskian formal semantics realise a philosophical conception of logic that doesn't prioritise justification and doesn't support a satisfactory formal treatment of justification (Gettier).

This conception of logic had a strong impact on AI/KR in its early days.

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A Solution:

Use alternative conceptions of logic along with their formal implementations for developing theoretical prototypes of KR systems.

Why HoTT? (1)

HoTT admits the constructive epistemically-laden proof-theoretic semantics intended by Martin-Löf's Type for MLTT (in a slightly modified form).

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Why HoTT? (2)

The cumulative *h*-hierarchy of types made explicit via the homotopical interpretation supports the distinction between propositional, set-level and higher-level types.

This distinctive feature of HoTT supports formal constructive representation of objects (of various levels) and propositions "about" these objects within the same framework. Each such object serves as a witness/truthmaker for proposition obtained via the propositional truncation of type where the given object belongs.

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Why HoTT? (3)

HoTT comprises a system of formal rules, which are interpreted as logical rules at the propositional *h*-level and as rules for object-construction at all higher levels.

This feature of HoTT supports representation various extra-logical procedures (such as material technological procedures) keeping track of the corresponding logical procedures at the propositional level of representation.

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Why HoTT? (4)

HoTT/MLTT is computationally implementable. Fragments of HoTT/MLTT have been implemented in proof-assistant Coq, program languages AGDA, LEAN and some other products.

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Why HoTT? (5)

HoTT-constructions admit intuitive spatial (homotopical) interpretations that may be used for facilitating human-computer interactions.

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The Morning Star is The Evening Star

Venus Homotopically http://philsci-archive.pitt.edu/12116/

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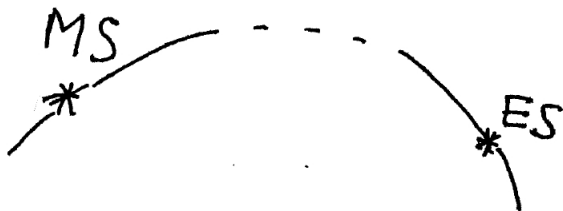
Remark 1

During the last decade KR technologies have been enriched with approaches based on the Big Data analysis, Machine Learning and artificial Neural Networks. According to a radical opinion, these new approaches make more traditional logical approaches based on the explicit representation of facts and rules hopelessly outdated and irrelevant. We disagree. Because of their possible unpredictable behaviour Neural Networks and other tools of the Big Data analysis can significantly enrich but not replace logical approaches and logical tools in KR.

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Remark 2

At the same time we agree that standard logical architectures and formal ontologies, which are presently used in KR, don't provide a sufficient theoretical background for KR because they have no epistemological content. In this paper we explained the relevance of epistemological considerations in logic and KR and then pointed to some recent advances in mathematical logic, more specifically discussing the Homotopy Type theory, that may allow to use logical approaches in KR more effectively.

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Thank You; Takk; Спасибо.