Restricting on solutions of optimization problems of information control in Network Generalization of Muddy Faces Puzzle

Denis Fedyanin
dfedyanin@inbox.ru

Institute of Control Sciences of Russian Academy of Science, Moscow
Higher School of Economics, Moscow
IEEE member

June 2, 2018

Model

Agents $N = \{1, ..., n\}$ are reasoning about theirselves. At any time an agent is in some state that we describe by pair $(x_i(t), r_i)$ for i-th agent at step t, where r_i is a contant parameter for an agent and could be one of q_i values, $x_i(t)$ is a binary parameter that describes a wish of an agent to be active or passive at time t. An agent makes this choice at time $t > 0$. Let $x_i(0) = 0$ for any i. (Fedyanin, 2017)

Fedyanin D.N. Threshold and Network generalisations of Muddy Faces Puzzle / Proceedings of the 11th IEEE International Conference on Application of Information and Communication Technologies (AICT2017, Moscow). M.: IEEE, 2017. V.1. . 256-260.

Agent's observations

All agents observe $x_i(t)$ when time is later than t and some agents can observe some r_i at time $t = 0$. We will specify who observes who by a square matrix $O = \{o_{ij}\}$, where o_{ij} means that i-th agent observes r_j.

Let ϕ be a public announcement at $t = O$. Let $w(\phi)$ be a set of possible worlds in which ϕ is true. Amount of eleminated worlds by ϕ is the only property of ϕ that we will use so we avoid difficulties with syntactic properties of certain public announcements (based on idea from Aumann 1999a).

Actions and activation algorithm

Let $x_i(t) = 0$ if there is j such that i-th agent does not know r_j at time t no matter by observations or by reasoning.
Let $x_i(t) = 1$ if there is no j such that i-th agent does not know r_j at time t no matter by observations or by reasoning.

Muddy Children Puzzle

Muddy Children Puzzle: $o_{ij} = 1$ iff $i \neq j$, and $o_{ii} = 0$ for any i and j. (Gierasimczuk N., Szymanik J., 2011) Classic: $n = 3$ (Littlewood, 1953)

The basic story of the puzzle is as follows (if it's required). Three children have muddy faces, and each can see the others faces, but not his own. A teacher announces to the children: "at least one of you has a muddy face". Then he asks: "Do you know whether your face is muddy or not? If so, raise your hand". No child raises a hand. Then, after some time, the teacher asks the same question, and again no child raises a hand. Some more time passes, and when asked the question a third time, each child raises his hand.

Optimization problem. Epistemic planning (Ghallab M. et al, 2004)

Let T_{max} be a number that for any $t > T_{max}$ and for any i holds $x_i(t) = 1$ an there is no $t < T_{max}$ that for any i holds $x_i(t) = 1$.
We will vary ϕ and O to get minimal value of T_{max}.
"Brute force" solution requires estimation of at least $2^{2^n-1}n^2$ variants so complexity is

$$O(2^{2^n}n^2)$$

.

Restrictions

We solve optimization problem for any given amount of unique eleminated by public announcement ϕ possible worlds

$$n_p \in [0; 2^{\prod_{I \in N} q_i - 1}]$$

and an amount of unique observations

$$n_o \in [0; n^2]$$

.

Analisys of epistemic planning for a single player

An amount of unique eleminated possible worlds (horizontal), an amount of unique observations (vertical).

-	0	1
0	-	1
1	1	1

Analisys of epistemic planning for 2 players

An amount of unique eleminated possible worlds (horizontal), an amount of unique observations (vertical).

-	0	1	2	3
0	-	-	-	1
1	-	2	-	1
2	-	1	1	1
3	-	1	1	1
4	1	1	1	1

Analisys of epistemic planning for 3 players

An amount of unique eleminated possible worlds (horizontal), an amount of unique observations (vertical).

-	0	1	2	3	4	5	6	7
0	-	-	-	-	-	-	-	1
1	-	-	-	2	2	2	-	1
2	-	2	-	2	2	2	-	1
3	-	2	-	1	1	1	1	1
4	-	2	2	1	1	1	1	1
5	-	2	2	1	1	1	1	1
6	-	1	1	1	1	1	1	1
7	-	1	1	1	1	1	1	1
8	-	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1

Analisys of epistemic planning for 4 players

An amount of unique eleminated possible worlds (horizontal), an amount of unique observations (vertical).

-	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	-	-	-	-	-	-	-	2	2	2	2	2	2	2	-	1
2	-	-	-	2	2	2	2	2	2	2	2	2	2	2	-	1
3	-	-	-	2	2	2	2	2	2	2	2	2	2	2	-	1
4	-	2	-	2	2	2	2	1	1	1	1	1	1	1	1	1
5	-	2	-	2	2	2	2	1	1	1	1	1	1	1	1	1
6	-	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1
7	-	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1
8	-	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1
9	-	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1
10	-	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1
11	-	2	2	1	1	1	1	1	1	1	1	1	1	1	1	1
12	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	-	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

The question is can we find analytical solution on this restricted optimization probems and explain and predict these crashes monotonity analytically.
The answer is yes, at least for special cases.

Example of optimal solution for activation on the first step

A matrix of observations.

0	1	1
0	1	1
0	1	1

Example 1 of optimal solution for activation on the second step

A matrix of observations.

0	1	1
0	0	0
0	0	0

Example 2 of optimal solution for activation on the second step

A matrix of observations.

0	1	1
1	0	0
1	0	0

Some references

1. Littlewood (1953). A mathematician's miscellany. London: Meuthen.
2. van Benthem J., van Eijck J., Gattinger M., Su K. Symbolic Model Checking for Dynamic Epistemic Logic.
3. Wang W. (eds) Logic, Rationality, and Interaction. Lecture Notes in Computer Science, vol 9394. Springer, Berlin, Heidelberg, (2015). 366- 378
4. van Benthem J., Erratum: "Rational dynamics and epistemic logic in games" // Int. Game Theory Rev., 09, 377 (2007). Pages 1345
5. Gierasimczuk N., Szymanik J.. A note on a generalization of the Muddy Children puzzle. Proceeding TARK XIII Proceedings of the 13th Conference on Theoretical Aspects of Rationality and Knowledge. Groningen, The Netherlands July 12 - 14, (2011). 257-264
6. Hintikka J., Reasoning about Knowledge in Philosophy: The Paradigm of Epistemic Logic . J. Symbolic Logic 53 (1988), no. 2, 663664
7. Aumann R.J. Interactive epistemology I: Knowledge// International Journal of Game Theory. August 1999, Volume 28, Issue 3, pp 263300
8. Ghallab M., Nau D.S., Traverso P.. Automated Planning: Theory and Practice. Morgan Kaufmann, 2004. 635 pages.

Thank you